https://www.linkedin.com/feed/update/urn:li:activity:6476027531152756736

$\lim_{n\to\infty}\left(2-\sqrt{2}\right)\left(2-\sqrt[3]{2}\right)\ldots\left(2-\sqrt[n]{2}\right).$

Solution by Arkady Alt , San Jose ,California, USA.

Let $P_n := \prod_{k=2}^{n}\left(2-\sqrt[k]{2}\right), n \in \mathbb{N}$. Noting that by AM-GM inequality

$\sqrt[n]{\frac{1}{2}} = \sqrt[n]{\frac{1}{2}\cdot 1\cdot 1\cdot\ldots\cdot 1} < \dfrac{\frac{1}{2}+(n-1)\cdot 1}{n} = \dfrac{2n-1}{2n}$ we obtain

$\sqrt[n]{2} > \dfrac{2n}{2n-1}$ and, therefore, $2-\sqrt[n]{2} < 2-\dfrac{2n}{2n-1} = \dfrac{2n-2}{2n-1}$.

Hence, $P_n < \prod_{k=2}^{n}\dfrac{2k-2}{2k-1} = \prod_{k=1}^{n-1}\dfrac{2k}{2k+1}$.

Since $\dfrac{2k}{2k+1} < \dfrac{2k+1}{2k+2}$ then $P_n^2 < \prod_{k=1}^{n-1}\dfrac{2k}{2k+1}\cdot\prod_{k=1}^{n-1}\dfrac{2k+1}{2k+2} =$

$\prod_{k=1}^{n-1}\dfrac{2k(2k+1)}{(2k+1)(2k+2)} = \prod_{k=1}^{n-1}\dfrac{k}{k+1} = \dfrac{1}{n} \Leftrightarrow P_n < \dfrac{1}{\sqrt{n}}$ and, therefore,

inequality $0 < P_n < \dfrac{1}{\sqrt{n}}$ by Squeeze Principle implies $\lim_{n\to\infty} P_n = 0$.